# PÁDOVÁ SIMULACE NA ŘEZU 2 (km 3,934)

***Trajectories of falling blocks along a slope***

The falling motion of a boulder along a rocky slope depends on many factors that are not easy to express numerically.

The trajectories of the boulders depend on the geometry of the slope, on the shape of the falling boulder and on its initial velocity at the moment of detachment from the slope, and also on the entity of the energy dissipated due to the impacts during the fall.

The falling boulders can slide, roll or bounce downstream depending on their shape, flattened or rounded, and on the gradient of the slope.

The energy dissipated due to impacts is generally different and varies with the characteristics of the motion and depends on the mechanical characteristics of the boulder and on the materials present along the slope (rock, soil, vegetation) that oppose in a different manner to the motion of the boulders.

In reality, however, it is practically impossible to determine precisely the contour of a slope and detect the shape of the different boulders that may detach.

In addition, the geometry of the slope and the nature of the outcropping materials undergo changes over time, sometimes sensitive, as a result of the alteration of the rock, of the accumulation of debris in the less steep areas and of the development of the vegetation.

Finally, it is practically impossible to model the motion of boulders fall in cases in which these shatter due to impacts, nor is it possible to identify the areas of the slopes where shatter occurs.

For the analysis of the falling trajectories we need to refer to very simplified models: the geotechnical design of the protection interventions must be, therefore, developed on the basis of a large numerical experimentation, making it possible to explore the different aspects of the phenomenon and recognize the main factors that affect the motion of fall in the particular situation in question.

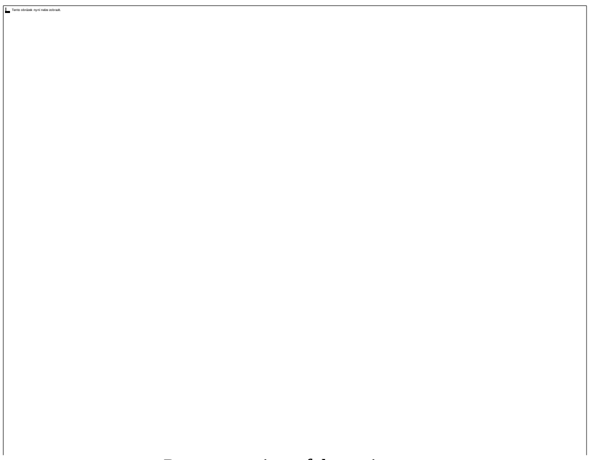
In more complex cases it might be necessary to calibrate the model on the basis of an analysis of trajectories detected by in situ cinematography following the collapse of the boulders.

***Lumped Mass computation model***

The assumptions of the Lumped Mass model are:

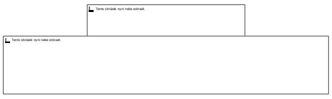
1) plan outline, the slope profile similar to a broken line consisting of straight line segments
2) point boulder and neglectable air resistance

In this case the trajectory of the boulder can be determined using the equations of motion of a rigid body



*Representation of the trajectory*

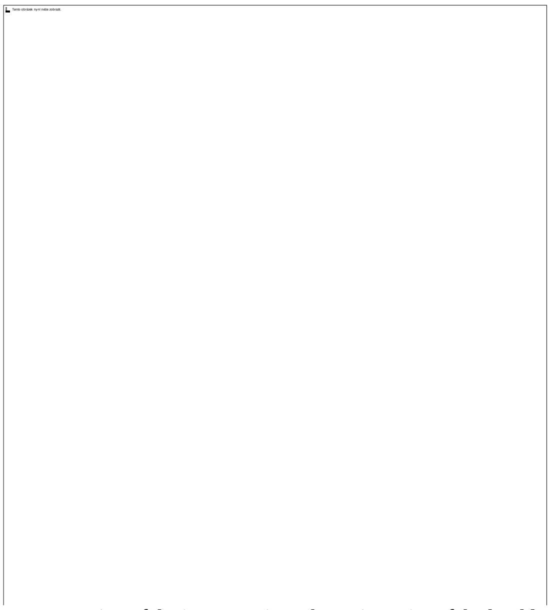
with reference to a system of orthogonal Cartesian axes the equations are:



where:

$v_x$ = horizontal component of the velocity of the boulder
$v_y$ = vertical component of the velocity of the boulder
$t$ = time
$g$ = acceleration of gravity
$x_0$ = abscissa of the point where the boulder is detached from the slope or impacts in the falling motion
$y_0$ = ordinate of the point where the boulder is detached from the slope or impacts in the falling motion

Along the x-axis the motion is uniform while along the y axis the motion is uniformly accelerated.
In this way the trajectory of the boulder motion is composed of a series of parabolas drawn between the point at which the detach takes place and the point at which the boulder collides on the slope for the first time, in the initial phase of motion, and between two successive impact points on the slope, or at the foot of the slope, later, to the stop point.
The coordinates of the impact points and velocity components are determined by solving the system between the equation (1) and the equation of the straight line representing the profile of the slope.
In practice we proceed from the point where the detachment of the boulder occurs and we resolve this system of equations considering in turn the different equations of the straight lines that contain the successive segments of the broken line up to finding the coordinates of a point, impact point, that belongs to the parabola that represents the trajectory and falls within of one of the segments of the broken line and is therefore also a point of the slope.
This point is the first impact point of the boulder on the slope. The procedure is repeated from that point to determine the next arc of the trajectory and a new impact point.



*Representation of the impact points, the trajectories of the boulder and the arrival and departure velocity vector at each bounce*

The loss of kinetic energy due to friction and impacts can be modeled by reducing the velocity of the falling boulder whenever this impacts on the slope.
In particular, indicating with vn and vt the components (normal and tangential) of the velocity before impact, after the impact $v'_n$, $v'_t$ can be calculated using the relationship:

where $R_n$ and $R_t$ are the restitution coefficients, variable in the range 0-1.

***CRSP computation method***

The *CRSP* model (*Colorado Rockfall Simulation Program*) has been developed by *Pfeiffer* and *Bowen* (1989) with the purpose of modeling the falling motion of boulders having the shape of spheres, cylinders or discs, with circular cross section in the vertical plane of the movement.

To describe the movement of the boulders the CRSP model applies the parabolic equation of motion of a body in free fall and the principle of conservation of the total energy.

The phenomenon of the impact is modeled using as additional parameters, compared to the *Lumped Mass* method, the roughness of the slope and the size of boulders.

In particular, the *CRSP* model assumes that the angle formed between the direction of the boulder and the profile of the slope varies according to a statistic that must be defined for each analyzed case. The model considers statistically also the results that mainly consist in the velocities and bounce heights, as compared to the surface of the slope, during the fall path.

So the model considers the combinations of movements of free fall, of bounce, rolling and slipping, which can vary depending on the size of the boulders and the roughness of the slope.

The reliability of the model was verified by comparisons between numerical results and the results obtained from in situ tests.

The description of the motion of free fall starts from a point in which the initial velocity is known and is decomposed into its horizontal and vertical components. The boulder is subjected to the movement of free fall until it collides with the surface of the slope.

From the intersection are obtained the coordinates of the impact point. The velocity vector of pre-impact V, forms an angle $\alpha$ with the slope.

*Representation of the impact phase: a) the angle of impact is defined as a function of the trajectory of the boulder; q) inclination of the slope; f) variation of the slope as a function of local roughness of the slope*

For each impact, the angle of the slope $\phi$ is varied randomly in a range of values between 0 and $\theta_{max}$. The value of $\theta_{max}$ depends on the roughness of the slope and on the size of the boulder and is determined by in situ measurements. Being R the radius of the boulder under consideration we have:

*Influence of the roughness of the slope on the path of the boulder: the ratio between the height of the asperities and the radius of the boulder*

The velocity that is obtained as a result of the impact is determined by the conservation equation of the total energy expressed as follows:

where:

| | |
|---|---|
| $R$ | = Radius of the boulder |
| $M$ | = Mass of the boulder |
| $J$ | = Moment of inertia of the boulder |
| $w_1$ | = Angular velocity before impact |
| $w_2$ | = Angular velocity after impact |
| $V_{t1}$ | = Tangential velocity before impact |
| $V_{t2}$ | = Tangential velocity after impact |

The function of friction *f(F)* is defined:

While the scale function *SF* is defined:

Where:

| | |
|---|---|
| $R_n$ | = *Normal restitution coefficient* |
| $R_t$ | = *Tangential restitution coefficients* |
| $R$ | = *Radius of the boulder* |

The terms f(F) and SF are obtainable through empirical expressions that are used to assess the kinetic energy dissipated in collisions between the boulder and the slope because of friction and impact.
The friction is primarily concerned with the dissipation of the energy produced by the tangential velocity, while the impact the energy produced by the velocity normal to the slope.

The tangential and angular post-collision velocities are related between them by the following equation:

which assumes that the boulders leave the contact with the slope rotating, regardless of the previous angular velocity. From (1) we obtain $V_{t2}$, while the normal post-collision velocity is obtained by the following empirical expression:

that will to take account of the fact, also verified experimentally, that the ratio between the normal post-impact and pre-impact velocities decreases with the increase of the normal pre-impact velocity itself.

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**BOULDER CHARACTERISTIC S**

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Boulder form Sphere

| | | |
|---|---|---|
| Density | 2700,0 | Kg/m3 |
| Elasticity | 50000,0 | kPa |
| Initial velocity in x | 4,0 | m/s |
| Initial velocity in y | -3,0 | m/s |
| Terminal limit velocity | 0,01 | m/s |
| Diameter | 1,2 | m |

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DESIGN VELOCITY OF BLOCKS

| | |
|---|---|
| Reliability coefficient for trajectory calculation | 1 |
| Quality coefficient of slope topography discretization | 1 |
| Velocity safety coefficient | 1 |

DESIGN BLOCK MASS

| | |
|---|---|
| Mass calculation coefficient | 1 |
| Survey precision coefficient | 1 |
| Survey precision coefficient | 1 |

DESIGN STRESSING ENERGY

| | |
|---|---|
| Energy amplifying coefficient | 1 |

BARRIERS

Safety coefficient to apply to the energy values MEL or SEL...1

| | | |
|---|---|---|
| Mass | 2442,903 | Kg |
| Weight | 2442,903 | Kgf |
| Moment of inertia | 351,778 | Kgxm2 |

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**Materials li st**

| N | Description | Coefficient normal restitution Rn | Coefficient of restitution tangential Rt | Roughness (m) | Frequency (m) | Friction angle (°) | |
|---|---|---|---|---|---|---|---|
| 1 | Solid rock | 0,9 | 0,8 | 0 | | | |
| 2 | Degraded rock | 0,7 | 0,7 | 0 | | | |
| 3 | Sand | 0,4 | 0,6 | 0 | | | |
| 4 | Rock detritus | 0,6 | 0,6 | 0 | | | |
| 5 | Fine debris | 0,32 | 0,82 | 0 | | | |
| 6 | Debris with vegetation | 0,29 | 0,8 | 0 | | | |
| 7 | Debris with shrubs | 0,3 | 0,7 | 0 | | | |
| 8 | Terrain or grass | 0,31 | 0,79 | 0 | | | |
| 9 | Paved surface | 0,4 | 0,9 | 0 | | | |

**SLOPE DA TA**

| N | X (m) | Y (m) | Material |
|---|---|---|---|
| 1 | -6,59 | 43,98 | Degraded rock |
| 2 | -4,26 | 41,28 | Terrain or grass |
| 3 | -0,77 | 37,77 | Solid rock |
| 4 | 1,97 | 34,71 | Solid rock |
| 5 | 4,63 | 31,21 | Solid rock |
| 6 | 8,48 | 25,6 | Solid rock |
| 7 | 10,81 | 22,11 | Solid rock |
| 8 | 14,87 | 13,49 | Solid rock |
| 9 | 15,86 | 8,41 | Solid rock |
| 10 | 16,54 | 2,0 | Degraded rock |
| 11 | 16,74 | 1,3 | Paved surface |
| 12 | 17,84 | 1,3 | Paved surface |
| 13 | 18,14 | 1,7 | Paved surface |
| 14 | 19,84 | 1,6 | Paved surface |

**IMPACT**

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**Throw no. 1 Xp=-2,73 m Yp=40,59 m**

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| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | -0,569 | 37,544 | 6,427 | -3,746 | 0,539 | 103,515 |
| 2,0 | 4,882 | 30,838 | 7,29 | -8,178 | 0,848 | 228,296 |
| 3,0 | 8,506 | 25,56 | 7,518 | -9,317 | 0,497 | 273,089 |
| 4,0 | 15,078 | 1,623 | 0,005 | 0,005 | 0,497 | 273,089 |

**Throw no. 2 Xp=-2,37 m Yp=40,23 m**

| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | -0,177 | 37,105 | 6,511 | -3,75 | 0,55 | 105,538 |
| 2,0 | 5,856 | 29,42 | 7,873 | -8,442 | 0,926 | 253,114 |
| 3,0 | 14,375 | 14,541 | 7,829 | -14,525 | 1,082 | 419,315 |

**Throw no. 3 Xp=-2,02 m Yp=39,88 m**

| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | 0,215 | 36,667 | 6,593 | -3,754 | 0,559 | 107,533 |
| 2,0 | 6,764 | 28,097 | 8,368 | -8,675 | 0,993 | 275,677 |

**Throw no. 4 Xp=-1,67 m Yp=39,52 m**

| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | 0,606 | 36,23 | 6,673 | -3,759 | 0,569 | 109,504 |
| 2,0 | 7,633 | 26,832 | 8,807 | -8,889 | 1,053 | 296,851 |

**Throw no. 5 Xp=-1,32 m Yp=39,17 m**

| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | 0,996 | 35,794 | 6,751 | -3,763 | 0,578 | 111,452 |
| 2,0 | 8,474 | 25,608 | 9,208 | -9,089 | 1,108 | 317,061 |

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**Throw no.  6    Xp=-0,97 m    Yp=38,82  m**

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| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | 1,385 | 35,359 | 6,827 | -3,767 | 0,588 | 113,377 |
| 2,0 | 9,339 | 24,314 | 9,372 | -9,543 | 1,165 | 338,955 |

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**Throw no.  7    Xp=-0,61 m    Yp=38,46  m**

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| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | 1,774 | 34,924 | 6,902 | -3,771 | 0,597 | 115,283 |
| 2,0 | 10,182 | 23,053 | 9,757 | -9,746 | 1,218 | 360,056 |

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**Throw no.  8    Xp=-0,24 m    Yp=38,08  m**

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| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | 2,215 | 34,383 | 6,285 | -4,878 | 0,615 | 119,112 |
| 2,0 | 7,136 | 27,555 | 7,732 | -8,191 | 0,783 | 240,93 |
| 3,0 | 15,171 | 1,614 | 0,005 | 0,005 | 0,783 | 240,93 |

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**Throw no.  9    Xp=0,09 m    Yp=37,71  m**

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| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|
| 1,0 | 2,617 | 33,854 | 6,423 | -4,908 | 0,632 | 122,851 |
| 2,0 | 7,994 | 26,306 | 8,129 | -8,443 | 0,837 | 260,689 |

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**Throw no.  10    Xp=0,42 m    Yp=37,33  m**

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| Ni | xi (m) | yi (m) | vx (m/s) | vy (m/s) | t (s) | E (kJ) |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| 1,0 | 3,016 | 33,329 | 6,554 | -4,936 | 0,648 | 126,502 |
| 2,0 | 8,857 | 25,036 | 8,283 | -8,911 | 0,891 | 280,989 |

**Defined types no.1**

| **Descr.** | **H (cm)** | **Thickness (cm)** | **Inclination (°)** | **E (kJ)** |
|---|---|---|---|---|
| 500kJ | 300,0 | 12,0 | 40,0 | 500,0 |

**Protection works inserted no.1**

| Descr. | Type | xb (m) | yb (m) | E (kJ) |
|---|---|---|---|---|
| 500kJ | 1,0 | 14,867 | 13,491 | 500,0 |

**Energy on barriers: Trajectory no.1**

| Descr. | Xb (m) | Yb (m) | Hp (m) | Vt (m/s) | Vd (m/s) | E (kJ) |
|---|---|---|---|---|---|---|
| 500kJ | 14,867 | 13,491 | 0,276 | 19,407 | 19,407 | 460,057 |

**Energy on barriers: Trajectory no.8**

| Descr. | Xb (m) | Yb (m) | Hp (m) | Vt (m/s) | Vd (m/s) | E (kJ) |
|---|---|---|---|---|---|---|
| 500kJ | 14,867 | 13,491 | 0,397 | 19,945 | 19,945 | 485,9 |

**(HpMax) Maximum height, (Vmax) Maximum velocity, (Emax) Maximum energy of the boulder upon the barrier**

| Descr. | Xb (m) | Yb(m) | HpMax (m) | Vmax (m/s) | Emax (kJ) |
|---|---|---|---|---|---|
| 500kJ | 14,867 | 13,491 | 0,397 | 19,945 | 485,9 |

**STATISTIC COMPUTATION S**

| | | |
|---|---|---|
| Maximum velocity | 20,619 | m/s |
| Minimum velocity | 9,206 | m/s |
| Average velocity | 12,925 | m/s |
| Mean standard deviation | 3,367 | m/s |
| **Maximum pre-impact energy** | **485,904** | **kJ** |
| Average pre-impact energy | 217,259 | kJ |
| Energy standard deviation | 112,423 | kJ |
| Average stop abscissa | 10,387 | m |
| Maximum abscissa reached | 15,171 | m |

% Stopped boulders

| X (m) | % Stopped boulders |
|---|---|
| 7,27 | 10 |
| 8,27 | 30 |
| 9,27 | 50 |
| 10,27 | 70 |
| 11,27 | 70 |
| 12,27 | 70 |
| 13,27 | 70 |
| 14,27 | 70 |
| 15,27 | 100 |
| 16,27 | 100 |
| 17,27 | 100 |
| 18,27 | 100 |
| 19,27 | 100 |